

Cosmology Model Information

The purpose of this model is to provide a simple and easy-to-use interface for the calculation of various cosmological values and to allow **non-scientists** to obtain a good spatial understanding of the expansion of the universe.

Specifically, the model focuses primarily on the “visible universe” and how that space has expanded since light first began to stream across universe at the age of approximately 380,000 years after the beginning of the universe (a.k.a. age of decoupling). These photons are known as the “last scattering surface” or “LS.”

The model will produce calculations of various distances and times when certain events occurred during the expansion of the universe. The model shows when the expansion of the universe began “accelerating” along with many other expansion rate values at different times during the evolution of the universe.

Further, the user may request a table of “z” values with many pages of calculations of different cosmological parameters at different times.

More knowledgeable users may move the “observer” from Earth today to points either in the past or in the future in order to see conditions that would have been (or will be) observed at that time.

The input form on the next page shows the required input data. When the model starts, the form is preloaded with the data shown on the example form. Each input item will be explained following the presentation of the input form.

Data Input Form

Default input values are shown below

Enter values - then click button

<input type="text" value="1091.0"/>	CMB_z (Now)
<input type="text" value="69.6"/>	H₀
<input type="text" value="0.286"/>	Omega_M
<input type="text" value="3.0"/>	z (redshift)

<input type="text" value="0.714"/>	Omega_{vac}
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Alternate Display Modes

Open Button
Sets Omega_{vac} = 0
thus, giving an open Universe.
[if the entered Omega_M < 1]

Flat Button
Sets Omega_{vac} = 1 - Omega_M - Omega_R
thus, giving a flat Universe.

General Button (default)
Uses Omega_{vac} entered.
Omega_R is set to $4.165E-5 / (H_0/100)^2$
Omega_K is set to $\Omega_K = 1 - \Omega_M - \Omega_{vac} - \Omega_R$

Detailed Input Data

The input data form shown is located on the left side of the output page.

H_0 (Now)

This input item refers to the redshift (z value) of the farthest light currently visible from Earth today (the observer's position) which is known as the "**Cosmic Microwave Background**" or **CMB**.

This value is somewhere between 1089 and 1100 depending on which study you are reading. The COBE mission came in about 1089; WMAP at around 1091; and Planck at ~ 1100 . The difference is very small, except for the determination of age at which the Last Scattering (LS) occurred.

H_0

This input item refers to the current value of the Hubble Parameter – also known as the **Hubble Constant**. The Hubble Constant is the expansion rate of the universe expressed in kilometers (km) per second (s) per megaparsec (Mpc) or "(km/s)/Mpc."

A single parsec is a very long distance – about 3.26 light years – or slightly more than 19,000,000,000,000 (19 trillion) miles. A megaparsec is 1,000,000 parsecs.

Parsec information: <https://en.wikipedia.org/wiki/Parsec>

This is a very important input item. Virtually every calculation requires knowledge of how fast space is expanding at a specific time and that expansion rate is calculated based in part upon this input item.

Once again, depending on which study you are reading, this value is expected to be around 70 with error bars of ± 4 .

See: https://en.wikipedia.org/wiki/Hubble%27s_law for detailed information.

Detailed Input Data (continued)

Ω_{M}

This input item is the **Matter Density Parameter (Omega M** or, using the Greek letter for omega (Ω), **Ω_{M}**) corresponding to today. This Ω_{M} value is input as a fraction and represents the **sum of normal (baryonic) matter and dark matter**. This fraction changes over time as the universe expands.

See: https://en.wikipedia.org/wiki/Lambda-CDM_model for detailed information.

Ω_{vac}

This input item is the **Dark Energy Density Parameter (Omega vac or Ω_{vac})** corresponding to today. This Ω_{vac} value is input as a fraction. This fraction changes over time as the universe expands.

Display Options for Output

This model is based upon Dr. Edward (Ned) L. Wright's work and many of the calculations may be found at:

<http://iopscience.iop.org/article/10.1086/510102>

There are certain parameters within the model for which assumptions are made as to their values today. Depending upon which output display option is chosen, these values will adjust accordingly.

The first parameter is the **Radiation Density Parameter** or **Omega r (Ω_r)**. Quoting Dr. Edward (Ned) L. Wright's paper:

“Another hidden aspect of the cosmology calculator is that it automatically sets the radiation density Ω_{rh^2} to the value appropriate for $T_0 = 2.72528$ K and three massless neutrino species, $\Omega_{rh^2} = 4.165 \times 10^{-5}$. Here $h = H_0/(100 \text{ km s}^{-1} \text{ Mpc}^{-1})$, and this factor includes a small (<1%) boost in the neutrino density due to a slight transfer to e^+, e^- annihilation energy into neutrinos (Hannestad & Madsen 1995).”

Using the default **H_0 of 69.6**, the value for **Ω_r** would be **0.00008598**.

The **Curvature Parameter (Ω_k)** is calculated by the following formula:

$$\Omega_k = 1 - \Omega_r - \Omega_m - \Omega_{vac}$$

This assumes the Total Density Parameter is 1 and the Curvature Parameter **Ω_k** is adjusted to the necessary value to achieve this value. Depending upon the choice of output, these parameters are handled in differing manners. Each will be explained below for each output option.

Using the default input fields yields a **Curvature Parameter** value as follows:

$$\Omega_k = 1 - \Omega_r - \Omega_m - \Omega_{vac}$$

$$\Omega_k = 1 - 0.00008598 - 0.286 - 0.714$$

$$\Omega_k = -0.00008598$$

Output Options

There are three basic and three more advanced options for the output display controlled by which button the user clicks. The output choice can dictate how the calculation of the Omega values is handled.



"Open" Output Option (Basic)

Choosing the **Open** Button causes Ω_{vac} to be set to zero.

The default settings for **Open** are then set to:

$$\text{CMBz} = 1091$$

$$H_0 = 69.6$$

$$\Omega_m = 0.28600000$$

$$\Omega_{vac} = 0.00000000$$

$$\Omega_r = 0.00008598$$

$$\Omega_K = 0.71391402$$

"Flat" Output Option (Basic)

Choosing the **Flat** Button causes Ω_K to be set to zero. Ω_{vac} is calculated by:

$$\Omega_{vac} = 1 - \Omega_m - \Omega_r$$

The default settings for **Flat** are then set to:

$$\text{CMBz} = 1091$$

$$H_0 = 69.6$$

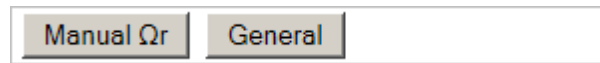
$$\Omega_m = 0.28600000$$

$$\Omega_{vac} = 0.71391402$$

$$\Omega_r = 0.00008598$$

$$\Omega_K = 0.00000000$$

Output Options (continued)



“General” Output Option (Basic)

Choosing the **General** Button causes Ω_k to be calculated by:

$$\Omega_k = 1 - \Omega_r - \Omega_m - \Omega_{vac}$$

The default settings for **General** are then set to:

$$\text{CMBz} = 1091$$

$$H_0 = 69.6$$

$$\Omega_m = 0.28600000$$

$$\Omega_{vac} = 0.7140000$$

$$\Omega_r = 0.00008598$$

$$\Omega_k = -0.00008598$$

“Manual Ωr” Output Option (advanced)

Choosing the **Manual Ωr** Button causes an input box to pop-up and allows to user to manually enter the Ω_r value. Again, Ω_k will be calculated by:

$$\Omega_k = 1 - \Omega_r - \Omega_m - \Omega_{vac}$$

The default settings for **Manual Ωr** are then set to:

$$\text{CMBz} = 1091$$

$$H_0 = 69.6$$

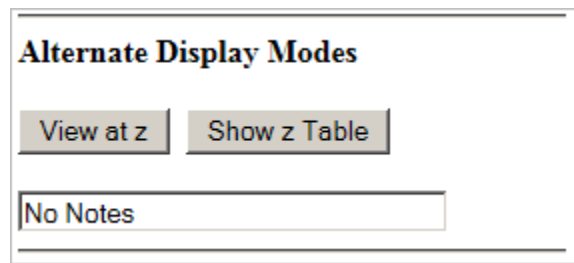
$$\Omega_m = 0.28600000$$

$$\Omega_{vac} = 0.7140000$$

$$\Omega_r = [\text{What user enters in Pop-up box}]$$

$$\Omega_k = 1 - \Omega_r - \Omega_m - \Omega_{vac}$$

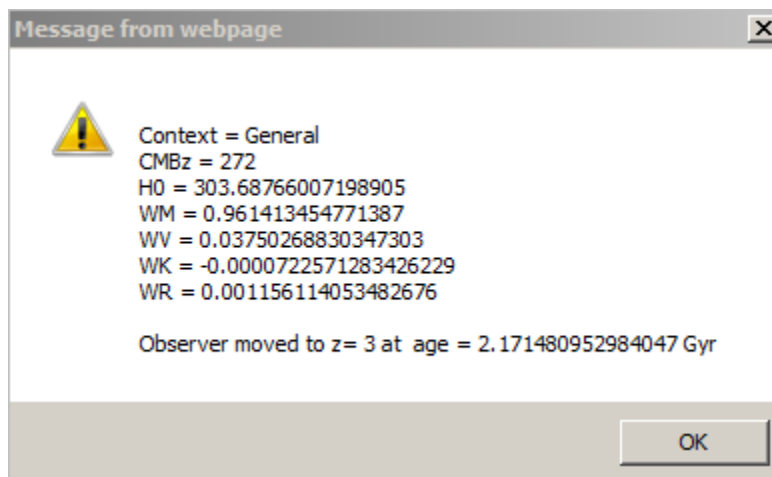
Output Options (continued)



"View at z" Output Option (Advanced)

Choosing the **View at z** Button causes the "observer" to move to the age of the previously-calculated z value and using the same output option choice (Open, Flat, General, Manual Ω_r) that was last used.

For example, using the default input values and the General output option, the age calculated for the default $z=3$ setting is 2.171480953 Gyr. Clicking the **View at z** button moves all calculations to the universe as it existed at that age. Initial settings would be:



Output Options (continued)

“Show z Table Output Option (Advanced)”

Choosing the **Show z Table** Button displays many pages of different calculations at various z values from 0.000 to CMBz (default = 1091).

These pages have detailed calculations that allow the user to see trends and also show special z values, including the Maximum LS-Photon proper distance z and the universe-begins-accelerating z.

The **z Table** will be calculated using the same output option choice (Open, Flat, General, Manual Ω_r) that was last used.

Use of the Omega Values

The reason that so much emphasis on the input data form is put upon getting the **Omega Values** properly set is that the calculation of the changing **Hubble Parameter** that defines the expansion rate of the universe at any point in time is governed by a combination of the current **Hubble Constant (H0)**, the **Scale Value z**, and the **Omega Values**. The formula for Hubble Parameter at a given z is:

$$E(z) = \sqrt{\Omega_r (1+z)^4 + \Omega_m (1+z)^3 + \Omega_k (1+z)^2 + \Omega_\Lambda}$$

$$H(z) = H_0 E(z)$$

The table on the next page shows the calculated **Hubble Parameter** and the **Omega Fractions** at various z values.

For more, see: [https://en.wikipedia.org/wiki/Distance_measures_\(cosmology\)](https://en.wikipedia.org/wiki/Distance_measures_(cosmology))

Hubble Parameter at various z values - Using Default Input Values:

For $H_0 = 69.600$ $\Omega_m = 0.28600000$ $\Omega_{vac} = 0.71400000$
 $\Omega_K = -0.00008598$ $\Omega_R = 0.00008598$

[General]	...Hubble...
.....	..Parameter. Ω_m Ω_v Ω_k Ω_r ...
.Redshiftat z....	.Fraction.	.Fraction.	..Fraction.	.Fraction.
....(z)..	..(k/s/Mpc).	..(at_z)..	..(at_z)..	... (at_z)..	.. (at_z)..
(Now) 0.00	69.60000	0.28600000	0.71400000	-0.00008598	0.00008598
0.02	70.20667	0.29828282	0.70171362	-0.00008791	0.00009147
0.04	70.83210	0.31061630	0.68937638	-0.00008979	0.00009712
0.06	71.47627	0.32298203	0.67700665	-0.00009160	0.00010292
0.08	72.13913	0.33536213	0.66462234	-0.00009335	0.00010889
0.10	72.82062	0.34773931	0.65224073	-0.00009504	0.00011499
0.20	76.50501	0.40902371	0.59093120	-0.00010247	0.00014756
0.30	80.64121	0.46805886	0.53186645	-0.00010824	0.00018293
0.40	85.21228	0.52355730	0.47633477	-0.00011243	0.00022035
0.50	90.19827	0.57472742	0.42512860	-0.00011519	0.00025917
0.70	101.32927	0.66292046	0.33685798	-0.00011723	0.00033880
0.80	107.43174	0.70006245	0.29967564	-0.00011692	0.00037883
0.90	113.86535	0.73292944	0.26676788	-0.00011597	0.00041865
1.00	120.61163	0.76189670	0.23775972	-0.00011452	0.00045810
1.50	158.49207	0.86176620	0.13768975	-0.00010363	0.00064768
2.00	202.22592	0.91469151	0.08457521	-0.00009166	0.00082495
2.50	250.83148	0.94411429	0.05497340	-0.00008109	0.00099340
3.00	303.68766	0.96141345	0.03750269	-0.00007226	0.00115611
3.50	360.37255	0.97211725	0.02663258	-0.00006494	0.00131511
4.00	420.57978	0.97903394	0.01955329	-0.00005887	0.00147163
4.50	484.07451	0.98366711	0.01476020	-0.00005377	0.00162645
5.00	550.66900	0.98686332	0.01140605	-0.00004945	0.00178008
6.00	692.56101	0.99074654	0.00721109	-0.00004255	0.00208493
7.00	845.26806	0.99280866	0.00484092	-0.00003731	0.00238773
8.00	1,008.03569	0.99394013	0.00340381	-0.00003320	0.00268926
9.00	1,180.25923	0.99455706	0.00248292	-0.00002990	0.00298992
Decimals drop here due to width limitations					
10.00	1,361.4	0.99487119	0.00186604	-0.00002719	0.00328996
25.00	4,954.2	0.99211583	0.00014092	-0.00001147	0.00775472
53.60	15,139.8	0.98384123	0.00001509	-0.00000542	0.01614909
90.00	32,750.3	0.97337126	0.00000322	-0.00000322	0.02662873
135.500	60,565.3	0.96058286	0.00000094	-0.00000212	0.03941831
181.000	93,857.0	0.94812505	0.00000039	-0.00000157	0.05187612
272.000	174,648.2	0.92415406	0.00000011	-0.00000102	0.07584684
363.000	272,266.3	0.90136532	0.00000005	-0.00000074	0.09863538
545.000	512,370.8	0.85900118	0.00000001	-0.00000047	0.14099928
1091.000	1,548,003.9	0.75284974	0.00000000	-0.00000021	0.24715047
